



Answer the following questions

Question 1

Solve the following differential equations:

i) $x \left(\cos \frac{y}{x} \right) (y dx + x dy) = y \left(\sin \frac{y}{x} \right) (x dy - y dx)$

ii) $y^2 \ln y = xyp + p^2$

iii) $x \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} - \frac{2}{x} y = x \ln x + 1$

iv) $(D^3 + 3D^2 + 2D)y = x^2 + 1$

v) $\sqrt{x} \frac{d^2 y}{dx^2} = x \left(\frac{dy}{dx} \right)^2$

vi) $\frac{dy}{dx} = \frac{x + y + 3}{2x + 2y + 1}$

Question 2

a) Test the convergence of the following series:

i) $\sum_{n=0}^{\infty} \left(\frac{n^2 + 2}{2n^2 + n + 1} \right)^n$

ii) $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

b) Draw and compute the Fourier series of the following function:

$$f(x) = \begin{cases} -x & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

c) Find the Laplace transform of the functions:

i) $f(t) = (t + 1)^2 e^{-t} \sin^2 t$

ii) $f(t) = \frac{2 - 2e^{-3t}}{t}$

Question 3

a) Solve the following system of simultaneous differential equations:

$$\frac{dx}{dt} + 5x + y = e^t, \quad \frac{dy}{dt} - x - 3y = e^{2t}$$

b) Evaluate :

$$\iint_D (2xy^2 + 2y \cos x) dA$$

where D is bounded by the curve $y = \sqrt{x}$, $x = 0$, and $y = 3$.

c) Find the orthogonal trajectories of the curve:

$$y^2 + 3x^2 = 2ax$$

Question 4

a) Find $L^{-1}\{F(s)\}$ by using the Convolution theorem:

$$F(s) = \frac{1}{s^3(s^2 + 1)}$$

b) Solve the initial value problem by using Laplace transform:

$$y'' - 2y' + 5y = -8e^{-t} \quad y(0) = 2, \quad y'(0) = 12$$

c) Find the inverse Laplace transform of the following functions:

$$i) s^2 F(s) + sF(s) - 6F(s) = \frac{s^2 + 4}{s^2 + s}$$

$$ii) F(s) = \tan^{-1}\left(\frac{1}{s}\right) + \ln \frac{s^2 + 9}{s^2 + 1}$$

Question 5

a) Find the mass and center of mass of the lamina in the shape of the region bounded by the graphs $y = x^2$ and $y = 4$ having mass density given by $\rho(x, y) = 1 + 2y + 6x^2$.

b) Evaluate the following integral:

$$\int_0^4 \int_0^4 \int_0^y \left(\frac{6}{1 + 48z - z^3} \right) dz dy dx$$

c) Draw the periodic function and then find its Laplace transform :

$$f(t) = e^t \quad 0 < t < 2\pi$$

d) Find the moments of inertia I_x, I_y, I_0 for the lamina that occupies the region D, where D is bounded by:

$$0 \leq x \leq 2, \quad -1 \leq y \leq 1; \quad \rho(x, y) = xy^2$$

With my best wishes

Dr. Eng. Rizk Masoud

This exam measures the following ILOs												
Question Number	Q 1-a	Q 1-a	Q 1-b	Q 5-c	Q4-c	Q5-b			Q3-b	Q 5-d		
	Q 2-a	Q 2-b	Q 2-c	Q3-c	Q3-a	Q4-b			Q4-a	Q5-a		
Skills	Knowledge & understanding Skills				Intellectual Skills				Professional Skills			